# Active vibration absorber for automotive suspensions: a theoretical study

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**Abstract:** The paper deals with the attenuation of mechanical vibrations in automotive suspensions using active vibration absorbers. The main features and performance of an automotive suspension featuring an active vibration absorber are assessed adopting a two-degree-of-freedom quarter-car model. The active vibration absorber is designed following a linear quadratic regulation (LQR) control law. A comparison of the proposed system with a suspension that uses a purely passive vibration absorber and a state-of-the-art active suspension is presented. The results of the numeric simulations show that active vibration absorbers could be effective in improving suspension handling and comfort performance. They can be as well as a possible alternative to standard active suspensions in terms of lower power consumption, simplicity and cost.

**Keywords:** vehicle handling and comfort; active automotive suspensions; active vibration absorbers; vehicle vertical dynamics.

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## 1 Introduction

Handling and ride comfort result in conflicting requirements for conventional suspensions, as the properties of spring and shock absorber are fixed and cannot be tuned according to the operating conditions of the vehicle. Therefore, in practice, passenger car suspensions are designed to achieve good tradeoffs. In order to improve handling and ride quality under various operating conditions, actively controlled suspensions have been studied in the past (Rajamani, 2012). In active solutions, an actuator is commonly introduced in place of or in parallel to the conventional spring and shock absorber to control the force applied to the sprung mass. Although active suspensions have been available for decades, they have not found widespread commercial application. It can be mainly explained when considering their large power consumption amounting approximately to two and half times the peak power required by the starter and three times the power drawn from the air-conditioning compressor. This is especially true for high-bandwidth versions (Chalasani, 1986a,b). Thus, commercial implementations can be found for low-bandwidth active suspensions (Sharp and Hassan, 1987), or for so-defined semi-active suspensions, where the damping of the shock absorber can be adjusted, for example, by using electro-rheological fluids (Hac and Youn, 1991; Redfield, 1991). An alternative solution to improve suspension performance is the use of dynamic vibration absorbers, which, in their simplest 'passive' form, typically consist of a reaction mass and a spring element with appropriate damping. Once the vibration absorber is connected to a primary target system (i.e., the sprung mass of the vehicle), it can absorb part of its vibrational energy (Zuo and Cui, 2013). These systems have proved effective in attenuating vibrations in various types of mechanical systems with relatively low cost (Inman, 2006), including civil engineering structures (e.g., bridges and buildings), internal combustion engines, ships and rotating machinery, automotive suspensions (Thomson et al., 1984) and seats (Kumaraswamidhas et al., 2012), and railway car bodies (Gong et al., 2013). Over the years, many design configurations of vibration absorbers have been developed along with optimal tuning rules for tonal and broadband applications (Koronev and Reznikov, 1993).

It is well known that the major drawback of purely passive vibration absorbers arises from the fact that the values of their physical parameters must be chosen according to a fixed value of the excitation frequency. If this value changes, the absorber is less effective, even resulting, for extreme cases, in amplified vibration levels. Therefore, for systems where the input is of a very specific frequency, the tuned mass is effective. Conversely, a passenger car is a typical system that is subject to a broadband random input, usually modelled as a white noise input. Therefore, a comprehensive modelling and optimisation is required to find the appropriate values of mass, stiffness and damping that minimise the vibrational level of the sprung mass (Zuo and Nayfeh, 2005). One possible approach is to employ 'active' dynamic vibration absorbers where, besides the passive elements, an actuator is introduced, which applies a control force obeying an appropriate control law to ensure tuning to the excitation frequency within a wide frequency band (Rivaz and Rohling, 2007). Thus, the extra DOF offered by an active vibration absorber may result in possible improved performance compared with a strictly passive solution. It should also be noted that an active design can allow the performance to be user-selectable. For example, if a softer or a harder ride behaviour is preferred, the weights in the performance index used for the controller design can be adjusted leading to different controller gains and, consequently, different system responses. Various configurations of active inertial absorbers are reviewed in Sun et al. (1995). An active configuration is proposed in Marques et al. (2001) for which the actuator exerts a force expressed as a linear combination of relative displacement, velocity and acceleration signals of the primary system. A fuzzy logic-based controller is presented in Yoshimura and Watanabe (2003) for an active suspension with dynamic absorbers. A new concept of active vibration absorber is originally presented in Olgac and Helm-Hansen (1995), named the Delayed Resonator, which uses partial state feedback with time delay. The delayed control logic can be implemented using position, velocity or acceleration measurements (Vyhlidal et al., 2014). In Millitzer et al. (2012), an active vibration absorber is proposed to reduce torsional vibrations in convertibles chassis. Semi-active vibration absorbers have also been investigated as a compromise between performance improvement and simplicity of implementation (Jalili, 2002; Koo and Ahmadian, 2004; Mo and Sunwoo, 2002).

In this work, the use of inertial absorbers is discussed for the automotive suspension case. The design of an optimal, fully active, high-bandwidth vibration absorber is described based on a two-DOF-quarter car (QC) model, which is simple but sufficiently detailed to capture many of the key suspension performance trade-offs, including ride quality (represented by sprung-mass acceleration), handling (represented by tyre deflection) and packaging (represented by suspension stroke). The goal is to control the motion of the sprung mass of the vehicle across a wider frequency range than it is possible with a passive mass damper and improve performance in terms of both handling and comfort. In order to obtain the appropriate control law, optimal control theory is used, based on the minimisation of a quadratic performance index that combines the previous performance measures with the control force by assigning adjustable weights. One should note that the present paper is intended as a proof of concept and, as such, it focuses on the analytical study and demonstration of the system via simulations. Experimental validation is out of the scope of the paper and it will be addressed in future research.

In principle, the study of vehicle vertical dynamics should consider the pitch and roll motions of the vehicle as well as the vertical (heave) motion (Reina and Foglia, 2013). However, it has been shown that the coupling between the pitch and roll and heave motions is not significant for typical passenger vehicles (Sayers and Karamihas, 1998). Therefore, in most cases, the quarter-car system is used for ride analysis (Reina et al., 2015) and active suspensions (Savaresi et al., 2010). As a simplified model for the quarter-car system the well-known two-DOF-QC model has been widely adopted, which is simple yet effective in describing the two dominant modes (sprung mass bouncing and wheel hopping). Recent studies by Mahaher and Young (2011) confirmed that a two-DOF-QC model provides a reasonable approximation of unsprung mass acceleration but overpredicts sprung mass acceleration magnitude, which is, however, a conservative condition.

The paper is organised as follows. The equations of motion of a QC suspension system with an active (or passive) vibration absorber are developed in Section 2. Optimal control theory is applied to design the controller of the active vibration absorber in Section 3. In Section 4, numeric simulations are presented to illustrate the main properties and effectiveness of the active vibration absorber, along with a comparison with a purely passive vibration absorber and a state-of-the-art active suspension. Conclusions are drawn in Section 5.

## 2 Vehicle modelling

Consider the design of an active mass damper based on a quarter-car model, as illustrated in Figure 1. The QC model considers both the vehicle sprung mass, $m_s$, as well as the unsprung mass, $m_n$, associated with the tyre-axle assembly. Their motion in the vertical direction can be described by two coordinates, $z_s$ and $z_n$, with origin at the static equilibrium positions of the sprung and unsprung mass, respectively. A passive suspension is initially assumed, where $k$ is the stiffness of the suspension spring, and $c$ is the damping coefficient of the shock absorber. The vertical tyre stiffness is $k_p$, whereas the tyre damping can typically be neglected. Subsystem ($m_d$, $c_d$, $k_d$) represents the vibration absorber, whose motion is described by the coordinate $z_d$. The actuator placed between the sprung and absorber mass applies a force $u$. If there is no control force, $u(t) = 0$, then the equations represent the vertical motion for a system with a passive vibration absorber. Overall, the QC model supported by the vibration absorber forms a three degree-of-freedom system, whose equations are given by:

$$m_s\ddot{z}_s + k(z_s - z_n) + k_d(z_s - z_d) + c(\dot{z}_s - \dot{z}_n) + c_d(\dot{z}_s - \dot{z}_d) = -u \tag{1}$$

$$m_n\ddot{z}_n - k(z_s - z_n) - c(\dot{z}_s - \dot{z}_n) + k_p(z_n - h) = 0 \tag{2}$$

$$m_d\ddot{z}_d + k_d(z_d - z_s) + c_d(\dot{z}_d - \dot{z}_s) = u, \tag{3}$$

where $h$ is the elevation of the road profile. By choosing the following variables as the states of the system

$$\begin{aligned} x_1 &= z_s - z_n \\ x_2 &= \dot{z}_s \\ x_3 &= z_n - h \\ x_4 &= \dot{z}_n \\ x_5 &= z_d - z_s \\ x_6 &= \dot{z}_d \end{aligned} \tag{4}$$

it is possible to derive the equations of motion in standard state-variable form:

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 0 & -1 & 0 & 0 \\ -\frac{k}{m_s} & -\frac{c+c_d}{m_s} & 0 & \frac{c}{m_s} & \frac{k_d}{m_s} & \frac{c_d}{m_s} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{k}{m_n} & \frac{c}{m_n} & -\frac{k_p}{m_n} & -\frac{c}{m_n} & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ 0 & \frac{c_d}{m_d} & 0 & 0 & -\frac{k_d}{m_d} & -\frac{c_d}{m_d} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \dot{h} \\ &+ \begin{bmatrix} 0 \\ -\frac{1}{m_s} \\ 0 \\ 0 \\ 0 \\ \frac{1}{m_d} \end{bmatrix} u, \end{aligned} \tag{5}$$

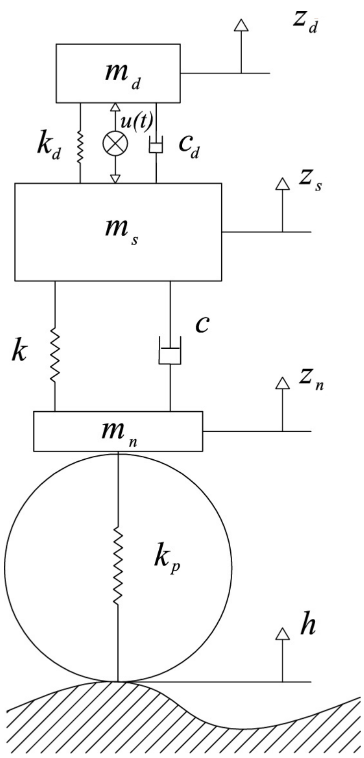
where $\dot{h}$ represents the vertical velocity of the tyre at the ground contact point, i.e., the disturbance input coming from the road, and hence $d = \dot{h}$. It is assumed that the ground-velocity input can be well modelled as a zero-mean white-noise input (Ulsoy et al., 2012). In compact matrix form, the equations of motion can be written as

$$\dot{x} = Ax + B_1 d + B_2 u, \tag{6}$$

where the state, disturbance, and input matrices are, respectively:

$$A = \begin{bmatrix} 0 & 1 & 0 & -1 & 0 & 0 \\ -\frac{k}{m_s} & -\frac{c+c_d}{m_s} & 0 & \frac{c}{m_s} & \frac{k_d}{m_s} & \frac{c_d}{m_s} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{k}{m_n} & \frac{c}{m_n} & -\frac{k_p}{m_n} & -\frac{c}{m_n} & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ 0 & \frac{c_d}{m_d} & 0 & 0 & -\frac{k_d}{m_d} & -\frac{c_d}{m_d} \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ -\frac{1}{m_s} \\ 0 \\ 0 \\ 0 \\ \frac{1}{m_d} \end{bmatrix}. \tag{7}$$

**Figure 1** Quarter-car model featuring a vibration absorber with active control force




Finally, at the end of this section some consideration can be drawn from equation (5) about the basic tasks of a vehicle's suspension system:

- *Isolation of the car body from road disturbances*: Comfort can be quantified by the vertical acceleration of the sprung mass $\ddot{z}_s$. A well-designed suspension aims to minimise sprung mass acceleration.
- *Keeping good road holding*: Road holding and handling performance are linked to the tyre ability to exchange tangential forces with the ground. Improved cornering, braking and traction are obtained if the variations in vertical loads are minimised. This is because the lateral and longitudinal forces generated by a tyre depend directly on the vertical load. Since the tyre radius depends on vertical forces, variations in tyre load can be directly related to vertical tyre deflection (i.e., $x_3$). Road holding and handling performance of a suspension can therefore be quantified in terms of the tyre deflection.
- *Suspension travel*: It is measured by the relative displacement of the sprung and unsprung mass (i.e., $x_1$). It defines the space required to accommodate the suspension spring movement between the bump and rebound stops, commonly known as the 'rattle space'.

## 3 Control design problem

Let us introduce the vector of the variables to be minimised, $z$, expressed in standard state-variable form

$$z = C_1 x + D_{12} u. \tag{8}$$

If the control design problem for the plant expressed by equations (6) and (8) is posed as that of minimising the variance of the output $z$, for the input $d$ being white noise, then this control design problem is known as the $H_2$ optimal control problem (Levine, 1996). It can be demonstrated that the solution to the $H_2$ optimal control problem is the same as the solution to the linear quadratic regulator (LQR) problem, where the controller is to be designed so that the following performance index is minimised

$$J = \int_0^\infty z^T z \mathrm{d}t = \int_0^\infty [x^T C_1^T C_1 x + 2x^T C_1^T D_{12} u + u^T D_{12}^T D_{12} u] \mathrm{d}t \tag{9}$$

for all initial conditions $x_0 = x(0)$. The solution to the LQR problem is

$$u = -\overline{k} x = -(D_{12}^T D_{12})^{-1} [B_2^T P + (C_1^T D_{12})^T] x, \tag{10}$$

where $P$ is positive semi-definite, and it is the solution to the Riccati equation:

$$A^T P + PA + C_1^T C_1 - (B_2^T P^T + D_{12}^T C_1)^T (D_{12}^T D_{12})^{-1} (B_2^T P + D_{12}^T C_1) = 0. \tag{11}$$

The effectiveness of an automotive suspension system is commonly measured in terms of ride quality, handling and packaging. The cost of the active control system should also be considered imposing a constraint that can be usually expressed as a function of the magnitude of the control force. Although it is desirable to maximise the system performance and to minimise the cost, both requirements cannot be achieved simultaneously. In order to reach a compromise of the two conflicting requirements, a performance index $J$ can be formulated with the appropriate weights. Many objective functions have been proposed in the literature to satisfy different design requirements in vibration control (Rajamani, 2012). In this research, the following quadratic performance index is proposed for the specific case of the control design of an active mass damper for automotive suspensions

$$J = \int_0^\infty [\ddot{z}_s^2 + \rho_1(z_s - z_n)^2 + \rho_2(z_n - h)^2 + \rho_3 u^2]\mathrm{d}t, \tag{12}$$

where the weighting factors $\rho_i$, $i = 1, \ldots, 3$, can be chosen so as to emphasise the relative variable of interest.

The performance index $J$ can be put into the standard matrix form of equation (9) and expressed in terms of root mean square of the variables of interest

$$\begin{aligned} J &= \int_0^\infty [x^T Q x + 2x^T N u + u^T R u]\mathrm{d}t \\ &= x_{\mathrm{rms}}^T Q x_{\mathrm{rms}} + \ldots + 2x_{\mathrm{rms}}^T N u_{\mathrm{rms}} + u_{\mathrm{rms}}^T R u_{\mathrm{rms}}, \end{aligned} \tag{13}$$

where

$$\begin{aligned} Q &= \begin{bmatrix} \frac{k^2}{m_s^2} + \rho_1 & \frac{k(c+c_d)}{m_s^2} & 0 & -\frac{kc}{m_s^2} & -\frac{kk_d}{m_s^2} & -\frac{kc_d}{m_s^2} \\ \frac{k(c+c_d)}{m_s^2} & \frac{(c+c_d)^2}{m_s^2} & 0 & -\frac{c^2+cc_d}{m_s^2} & -\frac{k_d(c+c_d)}{m_s^2} & -\frac{c_d^2+cc_d}{m_s^2} \\ 0 & 0 & \rho_2 & 0 & 0 & 0 \\ -\frac{kc}{m_s^2} & -\frac{c^2+cc_d}{m_s^2} & 0 & \frac{c^2}{m_s^2} & \frac{k_d c}{m_s^2} & \frac{cc_d}{m_s^2} \\ -\frac{kk_d}{m_s^2} & -\frac{k_d(c+c_d)}{m_s^2} & 0 & \frac{k_d c}{m_s^2} & \frac{k_d^2}{m_s^2} & \frac{k_d c_d}{m_s^2} \\ -\frac{kc_d}{m_s^2} & -\frac{c_d^2+cc_d}{m_s^2} & 0 & \frac{cc_d}{m_s^2} & \frac{k_d c_d}{m_s^2} & \frac{c_d^2}{m_s^2} \end{bmatrix}, \\ N &= \begin{bmatrix} \frac{k}{m_s^2} \\ \frac{(c+c_d)}{m_s^2} \\ 0 \\ -\frac{c}{m_s^2} \\ -\frac{k_d}{m_s^2} \\ -\frac{c_d}{m_s^2} \end{bmatrix}, \quad R = \frac{1}{m_s^2} + \rho_3. \end{aligned} \tag{14}$$

Along the lines of the solution discussed in equation (10), the solution to the optimal control problem that minimises the performance index $J$ is a state feedback law $u = -Gx$ where the feedback gain $G$ can be obtained by solving the following Riccati equation

$$A^T P + PA - (PB_2 + N)R^{-1}(B_2^T P + N^T) + Q = 0 \tag{15}$$

$$G = R^{-1}(B_2^T P + N^T). \tag{16}$$

## 4 Numerical results

In this section, the utility of inertial dampers to improve the vehicle vertical response is discussed using numerical simulations. First, it is studied the impact of passive and active vibration absorbers on standard automotive suspensions in both frequency and time domain. Then, the active absorber controlled through LQR optimisation is compared with a state-of-the-art active suspension.

### *4.1 Design of passive and active vibration absorbers*

The performances of a passive vibration absorber and an active counterpart, both mounted on the sprung mass of the vehicle, are studied and compared with a reference suspension, which is modelled as a passive, optimally damped QC system (i.e., with no vibration absorber). The parameters of the reference suspension are collected in Table 1. Note that the optimum value of damping in terms of acceleration, $c = c_{\text{opt}} = \sqrt{\frac{km}{2}}\sqrt{\frac{k_p+2k}{k_p}}$, is adopted as suggested in Genta and Morello (2009).

**Table 1** Parameters of the reference quarter-car suspension (with no vibration absorbers) used in the simulations, please refer to Figure 1 for more details

| | |
|---|---|
| $m_s$ | 454.5 kg |
| $m_n$ | 45.45 kg |
| $k$ | 27,625 N/m |
| $c_{\text{opt}}$ | 2801 Ns/m |
| $k_p$ | 221,000 N/m |

The vertical motion of a vehicle endowed with a passive vibration absorber can be described by equations (1)–(3) with no control force (i.e., $u(t) = 0$). For its tuning, a classical design method, proposed by Den Hartog (1947) for broadband applications of inertial dampers and known as the equal-peak method, is used. The following rule is stated to choose the stiffness and damping that minimises the amplitude of the motion of the (primary) sprung mass

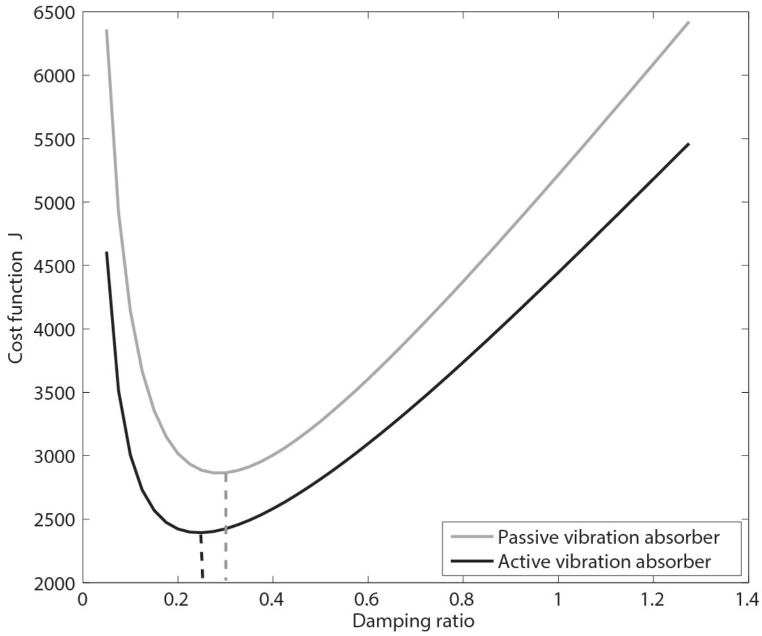
$$k_{d,\text{opt}} = k\frac{m_s m_d}{(m_s + m_d)^2}; c_{d,\text{opt}} = \frac{m_d}{m_s + m_d}\sqrt{\frac{3}{2}\frac{k_d m_s^2}{m_s + m_d}}. \tag{17}$$

However, other tuning alternatives can be used without altering the rest of the approach. For example, the vibration absorber could be tuned to match the vertical resonance of the abdominal cavity (4–7 Hz), which is known to be the frequency that human body has less tolerance for.

The mass of the passive vibration absorber, $m_d$, is chosen as 1/20 of the sprung mass. Finally, in order to set the appropriate damping of the suspension system endowed with passive vibration absorber, it is assumed that the suspension design problem can be formulated as a design-optimisation one, in which the objective is to select the damping coefficient, $c$, that minimises the performance index (12), considering typical ride characteristics (i.e., $\rho_1 = 5.0 \times 10^4$, $\rho_2 = 5.0 \times 10^3$, note that $\rho_3$=0 for a passive vibration absorber). A plot of $J$ vs. the damping ratio, $\xi = \frac{c}{2\sqrt{m_s k}}$, is shown in Figure 2 (solid grey line), and has a minimum at $\xi_{\text{opt}}$=0.3, which is chosen as the optimal damping of the suspension system coupled with the passive vibration absorber.

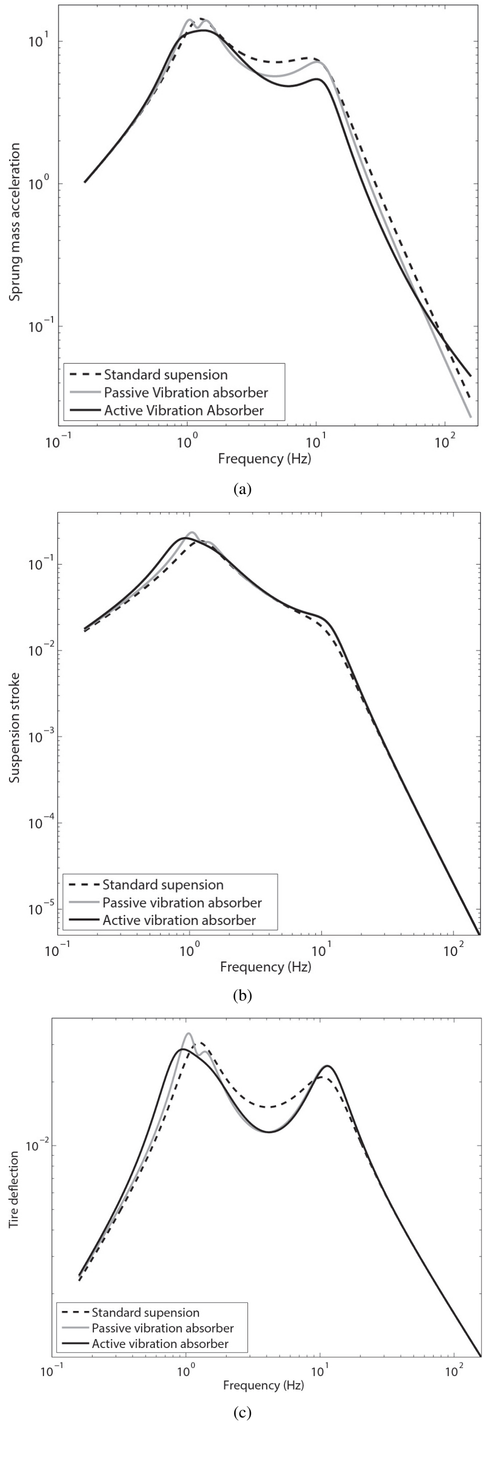
Similarly, the design of the active vibration absorber follows the same rationale of the passive embodiment, using the method proposed by Den Hartog to set $k_d$ and $c_d$, and an absorber mass $m_d$ equal to 1/20 of the sprung mass. The feedback gains of the control force are obtained through LQR optimisation (refer to Section 3), considering typical ride characteristics corresponding to the following weights in the performance index: $\rho_1 = 5.0 \times 10^4$, $\rho_2 = 5.0 \times 10^3$, $\rho_3 = 1.0 \times 10^{-5}$. Finally, the damping value of the suspension featuring an active vibration absorber resulted in $\xi_{\text{opt}}$=0.25, as shown in Figure 2 (solid black line).

**Figure 2** Performance index $J$ as a function of the damping ratio for a suspension system featuring a passive (solid grey line) and active (solid black line) vibration absorber. Note that the same weights in $J$ are used for both types of absorbers



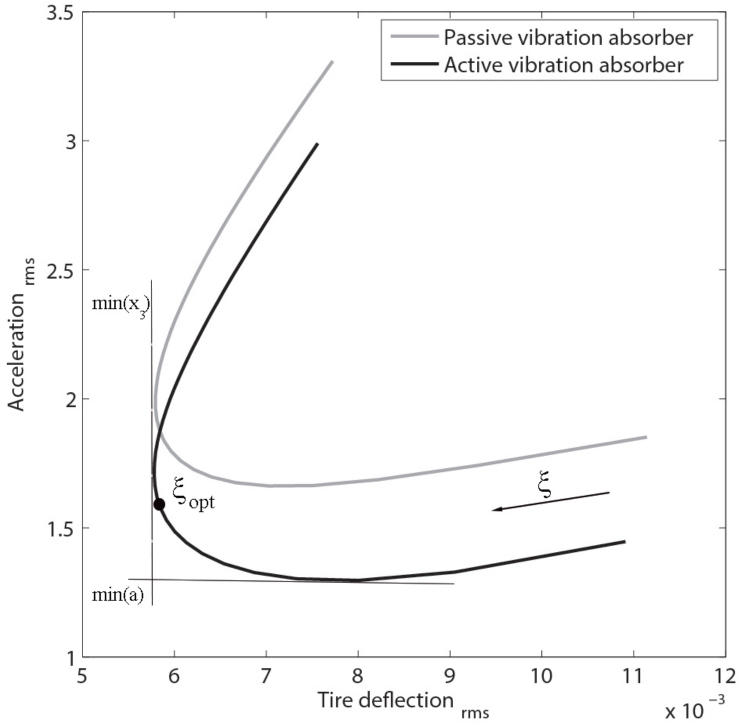

The frequency response of the sprung mass acceleration, suspension stroke, and tyre displacement are shown in Figure 3. In all graphs, the behaviour of the conventional suspension system is denoted with a dashed black line, whereas that of the passive vibration absorber is marked by a solid grey line, and that of the active absorber is denoted by a solid black line. It can be seen that the active vibration absorber outperforms both the conventional suspension and the passive absorber in terms of tyre deflection and sprung-mass acceleration, whereas its suspension deflection is somewhat worse than the conventional system. When comparing the three systems in terms of performance index $J$, the active system resulted in an improvement of about 20% with respect to the conventional system, and of about 15% with respect to the passive vibration absorber. Finally, the passive vibration absorber was found to improve the overall performance of about 7% with respect to the conventional system.

**Figure 3** Frequency response in terms of: (a) sprung mass acceleration; (b) suspension stroke; (c) tyre deflection for the three cases of: conventional suspension (dashed black line), passive vibration absorber (solid grey line), and active vibration absorber (solid black line)



It is also interesting to analyse the role of damping in the handling and comfort performance of a suspension endowed with vibration absorbers, as shown in Figure 4. The curves are the lower envelopes of the performance that may be obtained with passive (solid grey line) and active (solid black line) vibration absorbers as a function of the damping ratio $\xi$. As stated earlier, any improvement in comfort is accompanied by a worsening of handling and vice versa. The area between the horizontal and vertical tangents is the locus of the points defining optimal performance. The conditions leading to optimum comfort (minimum acceleration) and minimum handling (minimum tyre deflection) can be easily identified, as indicated in Figure 4 for the active vibration absorber. The first is obtained with a damping lower than the optimum damping ($\xi_{\mathrm{opt}}$) defined earlier, whereas the second for a damping value that is higher. The same behaviour holds for the passive vibration absorber. By comparing the two envelopes, it is apparent that the active absorber offers better performance than the passive absorber.

**Figure 4** r.m.s. value of the acceleration of the sprung mass vs. the rms value of the tyre deflection for a QC featuring an active and passive vibration absorber excited by a white noise ground-velocity input




## 4.2 Analysis in the time domain

The response of a suspension featuring an active vibration absorber is compared with that of a conventional suspension for the excitation of the same road-velocity input, modelled as a purely random variable (i.e., white-noise). Excitation variance is typically assumed as

$\sigma_d = 2\pi \cdot G_r \cdot V$, where $G_r$ is the road roughness coefficient and $V$ is the vehicle forward velocity. The International Standard Organization (ISO) defines a road classification scheme based on the value of $G_r$ (International Organization for Standardization, 1995), as shown in Table 2. For average road class (ISO class C), the road roughness coefficient value can be chosen as $64\times10^{-7}$ $m^2$ cycle/m for a travel speed of 25 m/s (Zuo and Nayfeh, 2003). Representative results for a 250 m-long road segment are shown in Figure 5, where the response of the two types of suspensions are evaluated in terms of acceleration (Figure 5(a)), and tyre deflection (Figure 5(b)), respectively. The active suspension shows a significant improvement over the standard system, with a reduction in the standard deviation of the sprung mass acceleration and tyre deflection of 21.1% and 14.5%, respectively.

It is worth looking at the specific behaviour of the active vibration absorber in terms of relative displacement with respect to the sprung mass (i.e., $x_5$), and power consumption, to evaluate its practical applicability. The mechanical power requirement can be obtained as $P(t) = u(t)(x_6 - x_2)$. Results for this simulation are shown in Figure 6, showing a maximum relative displacement of 8 cm and a power peak of 24 W, attesting to the feasibility of this approach.

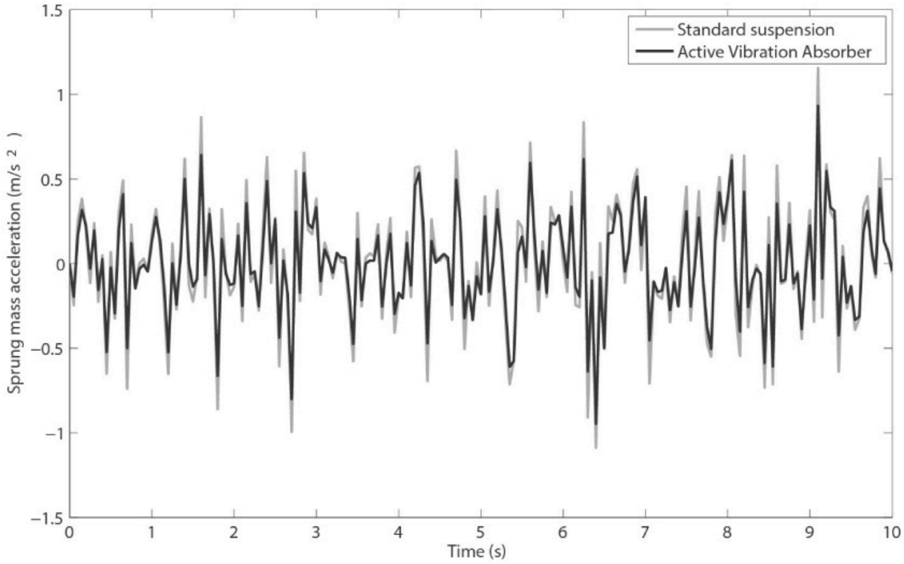
**Table 2** Road-roughness coefficients $G_r$ ($m^2 \cdot$ cycle/m)

| | *A* | *B* | *C* | *D* | *E* | | |
|---|---|---|---|---|---|---|---|
| *Road class* | *Very good* | *Good* | *Average* | *Poor* | *Very poor* | *F* | *G* |
| $G_r, \times10^{-7}$ | 4 | 16 | 64 | 256 | 1,024 | 4,096 | 16,384 |

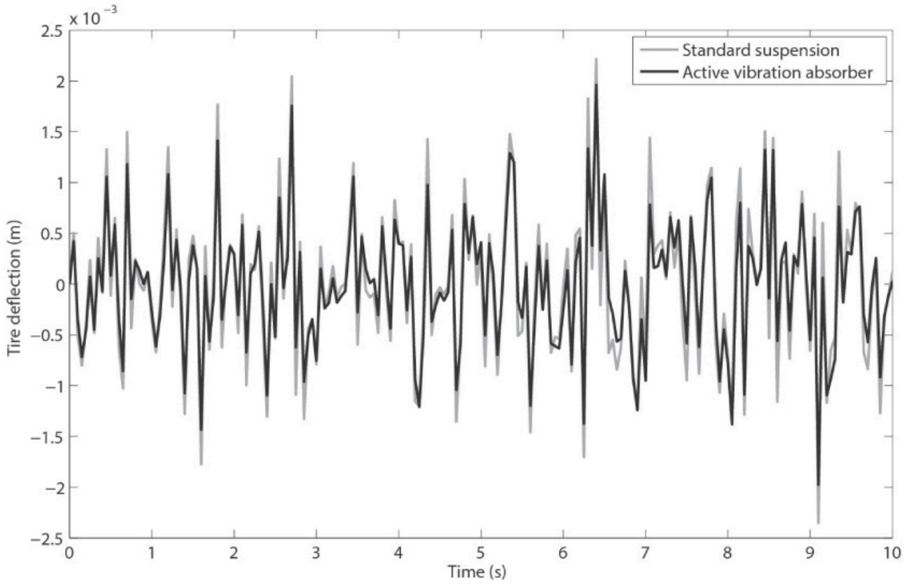
## *4.3 Active suspension vs. active vibration absorber*

It can be interesting to compare the performance of the active vibration absorber, presented in Section 4.1, with other active alternatives proposed in the literature. Specifically, a standard active suspension configuration is considered, where an actuator is placed in parallel with the passive elements of the suspension, according to the scheme of Figure 7. The relative equations of motion can be found, for example, in Butsuen (1989), who also developed the LQR formulation adopted in this study for the controller design of the standard active suspension. Using similar system parameters, it is possible to compare the frequency response of the two solutions in terms of sprung mass acceleration, suspension stroke and tyre deflection, as shown in Figure 8. One can see that the sprung mass acceleration and tyre deflection is considerably reduced at the sprung mass resonant frequency for the standard active control. However, no improvement is obtained at the unsprung mass resonant frequency compared to the active vibration absorber. Moreover, the suspension deflection is considerably increased at low frequencies. Although, the active suspension shows good performance, one limitation is its high power consumption with respect to active mass dampers as demonstrated in Figure 9. Thus, active vibration absorbers represent a possible alternative offering potential advantages compared to fully active suspensions in terms of limited energy, simplicity and cost.

**Figure 5** Response to a white-noise ground-velocity input of the active vibration absorber (solid black line) compared with a standard suspension (solid grey line) in terms of: (a) sprung mass acceleration, and (b) tyre deflection




(a)




(b)

**Figure 6** Response to a white-noise ground-velocity input in terms of relative displacement of the mass absorber with respect to the sprung mass, and power consumption

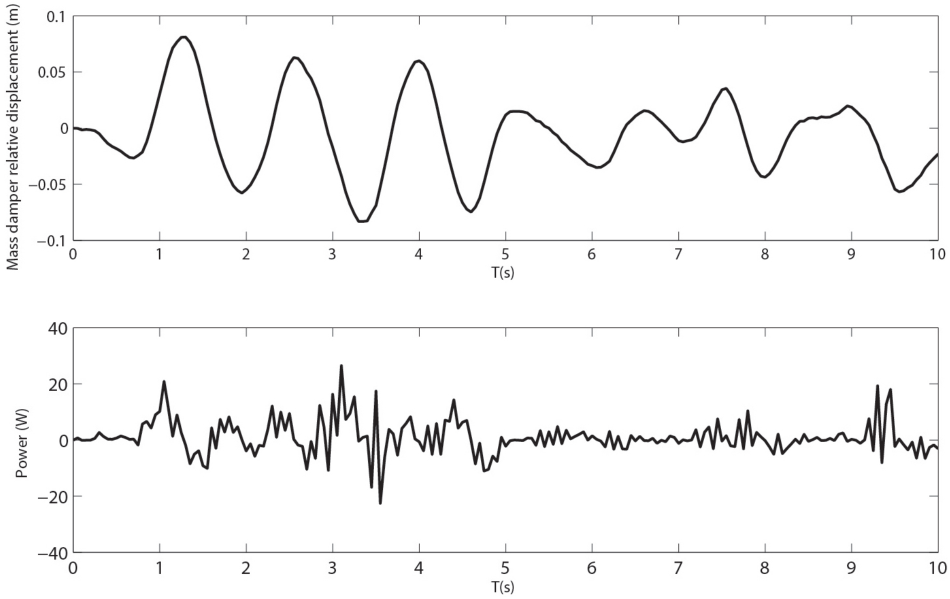



**Figure 7** Quarter-car model with standard active control

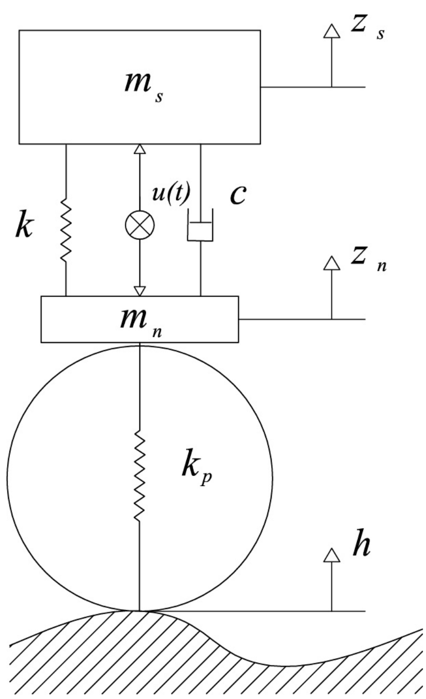
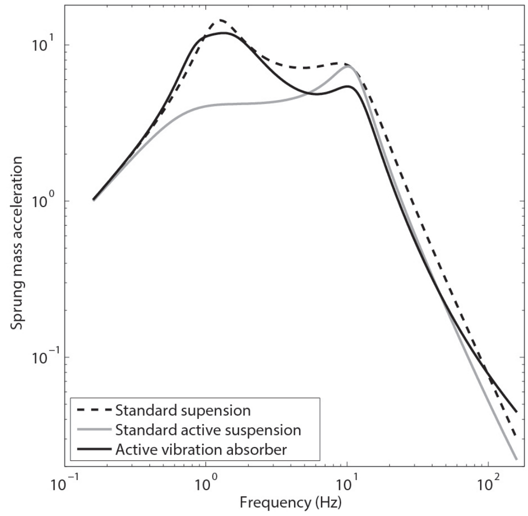
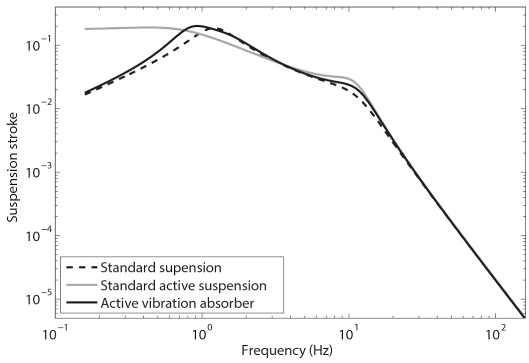


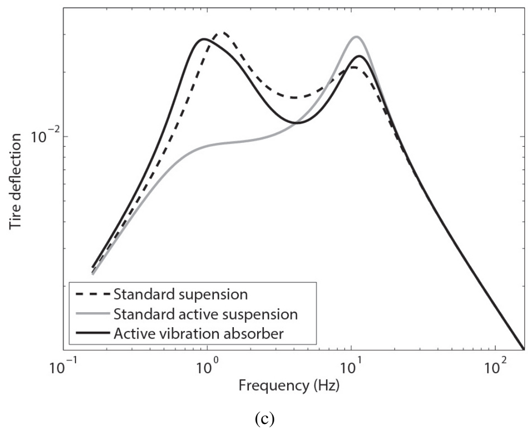
**Figure 8** Comparison of the active vibration absorber with a standard active suspension. Frequency plot of: (a) sprung mass acceleration; (b) suspension deflection, and (c) tyre deflection. Conventional suspension (dashed black line), active suspension (solid grey line), and active vibration absorber (solid black line).




(a)

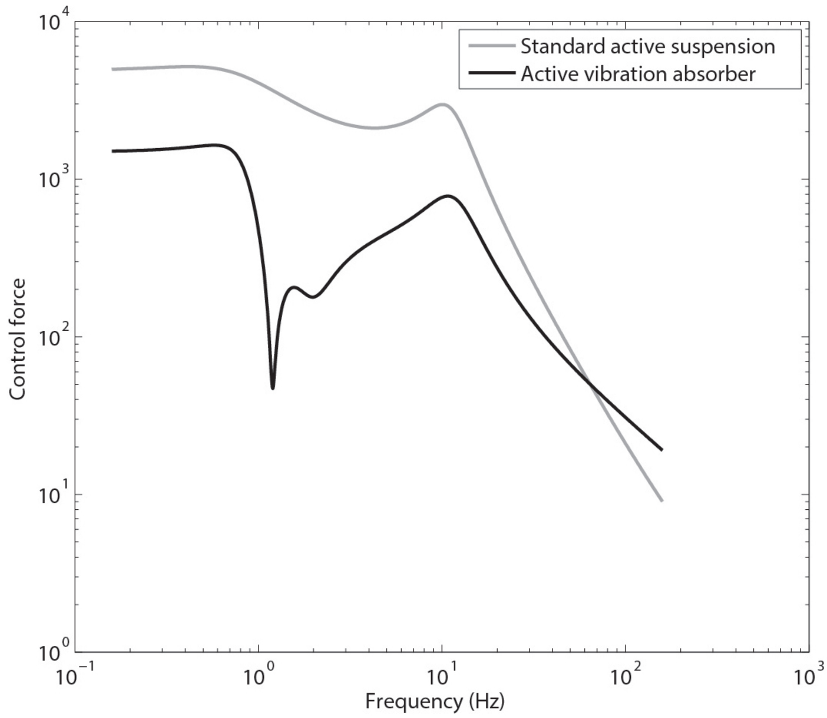



(b)




(c)

**Figure 9** Frequency plot of the control force $u$ for the standard active suspension (solid grey line), and active vibration absorber (solid black line)




## 5 Conclusions

This paper focused on the control of response of automotive suspensions using dynamic vibration absorbers. Both passive and active alternatives were studied in terms of design optimisation and performance evaluation. A LQR formulation was introduced for the optimal control design of active vibration absorbers. Simulation tests were presented, showing that active vibration absorbers could be effective in improving suspension performance of about 7% and 20% with respect to passive counterparts and conventional suspensions, respectively. They can be as well as a possible alternative to standard active suspensions in terms of lower power consumption, simplicity and cost.

While the paper focused on the feasibility of the system from a theoretical stand point, future research will be devoted to the experimental validation of the active vibration absorbers through laboratory test beds and real car prototypes. One of main challenges will be related to the practical implementation of the active system. We foresee the use of four independent vibration absorbers, one for each corner of the vehicle. The idea is to have four sealed cylinders located upright and rigidly attached to the sprung mass (chassis). Inside the cylinder it is a disc sandwiched between two coil springs and the unit is filled with damper oil. The reciprocating motion of the disc inside the cylinder will be controlled by an actuator. This solution seems possible considering the limited travel of the vibration absorber (within 8 cm). An electronic unit and a minimisation sensor suite, including accelerometers and vehicle level senders to estimate the relative displacements of the sprung and unsprung mass, will complete the package. As a final remark, it should be noted that one limitation of the proposed system is that the total weight of the vehicle will be increased by about 5%.

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## Nomenclature

| | |
|---|---|
| $m_s$ | Vehicle sprung mass |
| $m_n$ | Vehicle unsprung mass |
| $m_d$ | Vibration absorber mass |
| $z_s$ | Vertical displacement of the sprung mass |
| $z_n$ | Vertical displacement of the unsprung mass |
| $z_d$ | Vertical displacement of the vibration absorber |
| $k$ | Stiffness of the suspension spring |
| $k_d$ | Stiffness of the vibration absorber |
| $c$ | Damping of the suspension |
| $c_d$ | Damping of the vibration absorber |
| $u$ | Control force |
| $h$ | Road elevation |
| $d$ | Disturbance to the system |
| $x$ | State variable |
| $A$ | Sate matrix |
| $B_1$ | Disturbance matrix |
| $B_2$ | Input matrix |
| $J$ | Performance index |
| $\rho_i$ | Weighting factors of the performance index |
| $G$ | Feedback gain |
| $\zeta$ | Damping ratio |
| $V$ | Vehicle longitudinal speed |
| $G_r$ | Road roughness coefficient |
| $P$ | Solution of Riccati equation |