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Adaptive Kalman Filtering for GPS-based Mobile Robot Localization

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Abstract —Kalman filters have been widely used for navigation in mobile robotics. One of the key problems associated with Kalman filter is how to assign suitable statistical properties to both the dynamic and the observational models. For GPS-based localization of a rough-terrain mobile robot, the maneuver of the vehicle and the level of measurement noise are environmental dependent, and hard to be predicted. This is particularly true when the vehicle experiences a sudden change of its state, which is typical on rugged terrain due, for example, to an obstacle or slippery slopes. Therefore to assign constant noise levels for such applications is not realistic. In this work we propose a real-time adaptive algorithm for GPS data processing based on the observation of residuals. Large value of residuals suggests poor performance of the filter that can be improved giving more weight to the measurements provided by the GPS using a fading memory factor. For a finer gradation of this parameter, we used a fuzzy logic inference system implementing our physical understanding of the phenomenon. The proposed approach was validated in experimental trials comparing the performance of the adaptive algorithm with a conventional Kalman filter for vehicle localization. The results demonstrate that the novel adaptive algorithm is much robust to the sudden changes of vehicle motion and measurement errors.

Keywords: *GPS, mobile robot localization, adaptive filtering, fuzzy logic*

I. INTRODUCTION

In the last few years, mobile robots are increasingly been employed in high-risk, rough terrain situations, such as reconnaissance, safety, and rescue applications. They are required to explore larger and larger areas, performing difficult tasks, while preserving, at the same time, their safety. The success of the planned tasks is tightly connected with the vehicle's ability to self-localize, i.e. to produce accurate estimates of its position and attitude. This primarily requires advanced sensing and perception capabilities. For outdoor applications, absolute localization based on GPS is commonly available for mobile robots [1]. In order to enhance localization accuracy and robustness, Kalman filtering has been widely applied in GPS-based localization [2]. However, a conventional Kalman filter fails to estimate accurately the turning points of a mobile robot. It typically provides poor results for sudden changes in the vehicle's state [3]. The Kalman filtering is an optimal recursive estimation method that has been widely applied in real-time processing of incomplete and noisy measurements. It estimates the state of a dynamic system using two different models, namely the dynamic and the observation model. The dynamic model describes the behavior of the state vector, while the observation model establishes the relationship between measurements and the state vector. Both models are associated with statistical properties to describe the accuracy of the models. For many applications, the statistic noise levels of the model are given before the filtering process and will maintain unchanged during the whole recursive process. Commonly, this *a priori* statistical information is determined by test analysis and certain knowledge about the observation type beforehand. If such *a priori* information is inadequate to represent the real statistic noise levels, Kalman estimation is not optimal and may produce unreliable results, sometimes even leads to filtering divergence [4]. Such is the case when the vehicle comes to a sudden stop or steers with small turning radius. Those manoeuvres are hardly predictable by the filter. Therefore, a system with constant noise variances is inadequate to satisfy all situations and difficult to design. Specifically, one typical problem with vehicle localization using Kalman filter is the so called ”over shooting” problem, i.e. the dynamic model keeps estimating the robot position according to the previous trend while the vehicle is actually turning to another direction. Adaptive filtering tries to determine the statistical parameters of the dynamic system based on the behavior of the vehicle during data processing, and it has been paid much attention in Kalman filtering theory [5], [6]. Different adaptive Kalman filtering algorithms have been studied for surveying and navigation applications. Mohamed and Schwarz [4] applied adaptive Kalman filters for the integration of GPS and inertial navigation system (INS). Wang et al [7] applied a simplified adaptive algorithm in kinematic GPS positioning. Hu et al [3], developed two adaptive algorithms using GPS, one based on the fading memory and the other based on the variance estimation. In this work, we propose an adaptive filter for vehicle localization based on GPS data similar in principle to [3], using the fading memory approach in conjunction with fuzzy logic. The fading memory approach aims at estimating a scale factor to increase adaptively the predicted variance components of the state vector. This is performed by observing the residuals of

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the filter which reflect the discrepancy between the predicted measurement of the filter with the actual one. A residual of zero means that the two are in complete agreement and the filter is working properly. We found that fuzzy logic is well suited for this task and developed a divergence indicator which receives as input the size of the predicted residuals and issues as output a corrective scale factor for the estimate error covariance. If there is no divergence, the conventional Kalman filtering is used. Otherwise the adaptive algorithm is applied. The adaptive algorithm for GPS-based localization was experimentally validated using a rover named Dune, built at the Space Robotics Laboratory of the University of Tohoku. The vehicle, shown in Fig. 1, features a four wheel drive independent steering system that enhances maneuverability by enabling maneuvers such as turn-on-the-spot and crab motion. It also employs a passive rocker-type suspension system based on a differential gearbox to connect the two side links of the rover improving the ability of the vehicle to climb up obstacles and traverse uneven terrain while ensuring good traction performance. Section II describes in detail the adaptive kalman filer algorithm. Experimental results, obtained from Dune operating with different speeds and terrain conditions, are presented in Section III demonstrating that the positing accuracy with the adaptive approach is significantly better than the conventional Kalman filtering, especially when the vehicle stops and starts repeatedly, and turns around the corners.

II. The Adaptive Kalman Filtering Algorithm

Kalman filtering is a well known technique for state and parameter estimation. It is a recursive estimation procedure that uses sequential sets of measurements. In recent years Kalman filter based localization has became common practise in the robotics literature. The Kalman filter addresses the general problem of estimating the state $x \in \Re^n$ of a discrete-time controlled process that is governed by the linear stochastic difference equation

$$x_{k+1} = Ax_k + Bw_k \tag{1}$$

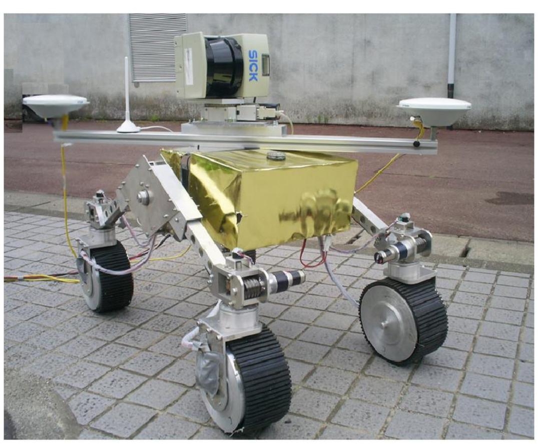

Fig. 1. The rover Dune

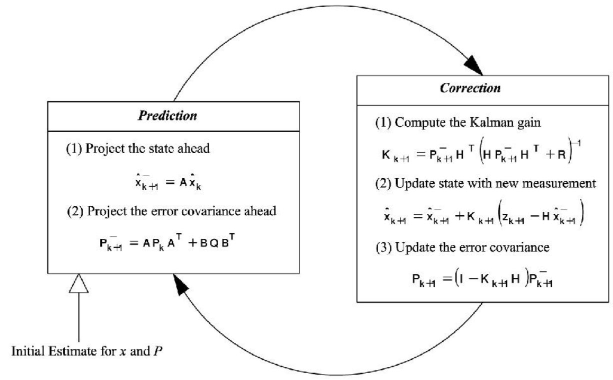

Fig. 2. Kalman filter operation diagram, adapted from [8]

with a measurement $z \in \Re^m$

$$z_{k+1} = Hx_{k+1} + v_{k+1} \tag{2}$$

The random variables w_k and v_k represent the process and measurement noise, respectively. They are assumed to independent of each other, white, with normal probability distributions, being Q and R the process and measurement noise covariance, respectively. Kalman filtering estimation can be expressed as

Prediction:

$$\hat{x}^-_{k+1} = A\hat{x}_k \tag{3}$$

$$P^-_{k+1} = AP_kA^T + BQB^T \tag{4}$$

Correction:

$$K_{k+1} = P^-_{k+1}H^T(HP^-_{k+1}H^T + R)^{-1} \tag{5}$$

$$\hat{x}_{k+1} = \hat{x}^-_{k+1} + K_{k+1}(z_{k+1} - H\hat{x}^-_{k+1}) \tag{6}$$

$$P_{k+1} = (I - K_{k+1}H)P^-_{k+1} \tag{7}$$

where $\hat{x}^-_{k+1}$ is the predicted state vector, P^-_{k+1} is the variance matrix for $\hat{x}^-_{k+1}$, K_{k+1} is the gain matrix, $\hat{x}_{k+1}$ is the updated state vector, and P_{k+1} is the updated error covariance estimate [8]. The prediction equations are responsible for projecting forward in time the current state and error covariance estimates to obtain the *a priori* estimates for the next time step. The correction equations are responsible for the feedback, i.e. for incorporating a new measurement into the *a priori* estimate to obtain an improved *a posteriori* estimate. A complete picture of the Kalman filter cycle with equations is shown in Fig. 2. The Kalman filtering estimation at a given time k can be considered as a weighted combination between the new measurement (observation model) and the predicted state vector based on the dynamic model and all previous measurements. If too much 'weight' is assigned to the dynamic model, the information from measurements is trusted less and less and this could lead to poor position estimation and even to possibly divergence of the filtering process. The approach envisaged by the fading memory is based on applying a scale factor M to the *a priori* estimate error covariance to deliberately increase the variance of the predicted state vector and thus resulting in more 'weight' given to the actual measurements

$$\bar{P}_{k+1}^{-} = M(AP_kA^T + BQB^T) \quad (8)$$

The main difference between different fading memory algorithms is on how to calculate the scale factor M. One simple approach is to assign the scale factor as a constant, but this leads to some drawbacks. For example, as the filtering proceeds, the accuracy of the pose estimation will decrease because the effects of old data will become less and less. The optimal solution is to use a variable scale factor that will be determined based on the dynamic and observation model accuracy. In this paper, we propose a fading memory algorithm using fuzzy logic to adjust adaptively the scale factor based on the size of the predicted residuals. The predicted residual vector, or innovation vector, represents the difference between the actual measurement and the predicted one and is expressed by

$$\hat{I}_{k+1} = z_{k+1} - H\hat{x}_{k+1}^{-} \quad (9)$$

For an optimal kalman filtering, the innovation vector should be a zero mean white noise [9]. Therefore the performance of the kalman filter can be measured using the value of the innovation vector. Our hypothesis is that deviation of the innovation vector from zero by more than a certain value suggests reduction in performance of the filter that can be corrected with a scale factor applied to the covariance matrix. For a finer gradation of the scale factor, we adopted fuzzy logic that uses rules to map from inputs to outputs, and defined what we call a Divergence Indicator (DI). The triangular membership functions used in the DI, i.e., the curves that map each point in the input space to a membership value or grade between zero and one, usually referred to as the normalization process, are shown in Fig. 3. The fuzzy inference system uses one input and one output. Input is the innovation vector. The output is the scale factor that quantifies the degree of confidence we have that divergence is occurring. Normalization is performed using the *if-then* rule set shown in Table I whereas the thresholds for the memberships function were experimentally-determined to give the best performance over other alternatives. Defuzzification is executed using the center of gravity method [10]. In our approach, we only try to estimate the variance factor of the dynamic model, as GPS measurement noise can be assigned to a reasonable level based on the type of GPS receiver employed.

Rule	Input: I_{k+1}	Output: Scale Factor M
1	Small	Low
2	Large	High

TABLE I
FUZZY LOGIC RULES FOR THE DI

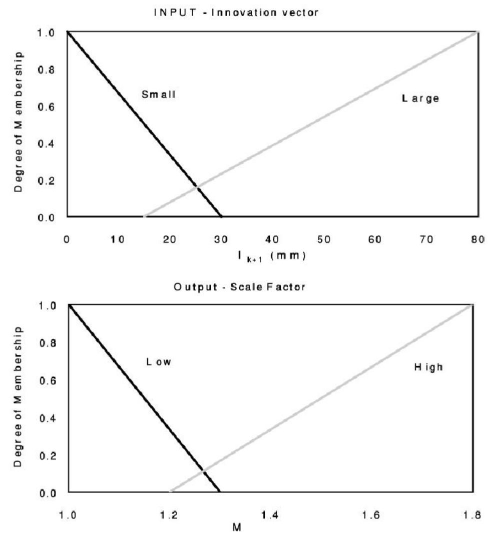

Fig. 3. Membership functions of the DI

III. EXPERIMENTS

The adaptive algorithm for GPS-based mobile robot localization was experimentally validated on the rover Dune. For the experiments, we used two Trimble 5700 dual frequency GPS receivers. One receiver was set as a reference station. Another GPS receiver was installed on the top of the rover Dune (see Fig. 1). Both dual frequency pseudorange and carrier phase measurements were collected during the test. A constant velocity model was adopted as the dynamic model for the Kalman filter. The state vector consists of the geocentric coordinate (X, Y, Z) and velocity $(\dot{X}, \dot{Y}, \dot{Z})$. The accelerations are considered as the dynamic model noise. The double difference pseudoranges were formed as the observation, and therefore, the receiver clock bias errors were not modeled in data processing. We focused on two types of experiments. In the first experiment, the rover was driven along a short distance, L-shaped path. The travel velocity was slow and the

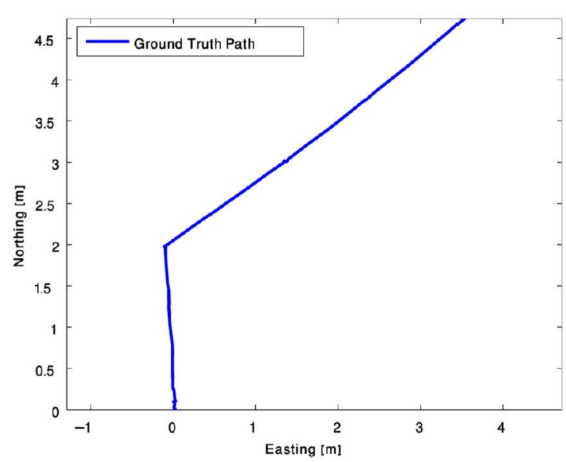

Fig. 4. Path of the rover

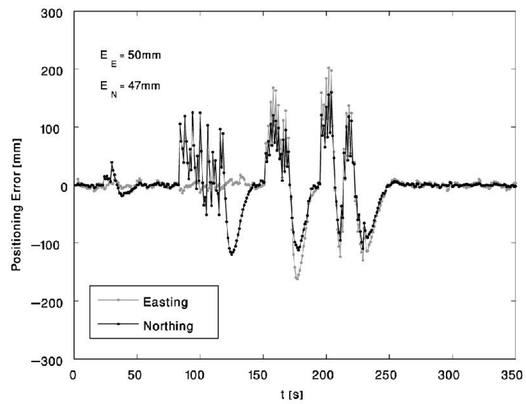

Fig. 5. Positioning error with conventional Kalman filter (σ_1=0.1 m/sec^2)

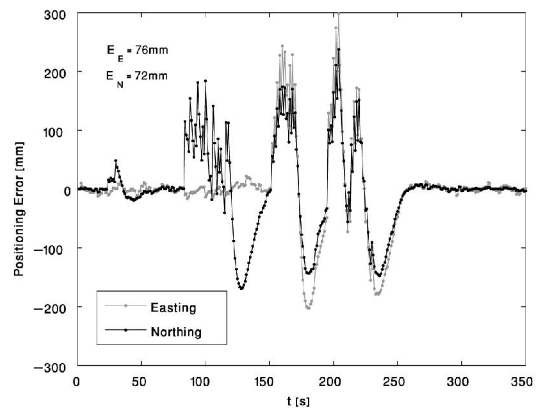

Fig. 6. Positioning error with conventional Kalman filter (σ_2=0.05 m/sec^2)

terrain flat without any significant variations in altitude. In the second experiment, the rover was commanded to follow a long distance, closed path. In this test the speed was moderate and the vehicle had to negotiate a steep slope in both the downward and upward directions.

A. L-path experiment

The rover was remotely controlled with a joystick along an approximately 2 m$\times$4 m, L-shaped path with a travel speed of about 5 cm/s. As an initial step, the reference trajectory was obtained with kinematic GPS positioning using carrier phase measurement in RTK GPS mode (accuracy of 10 mm + 1 ppm RMS, [11]), which we considered as the ground truth in our experiment. Then, the L1 C/A code data (accuracy of 0.25 m + 1 ppm RMS) were processed using different Kalman filters. In order to validate the performance of the adaptive kalman filter in detecting changes in Dune's state, the rover was repeatedly stopped and started along the way, and also performed the heading change with a turn-on-the-spot maneuver. Figure 4 shows the path followed by the rover as estimated by the RTK-GPS. The conventional Kalman filtering method was firstly used to process the DPGS data. Two different dynamic noise levels had been chosen to analyze the influence of noise

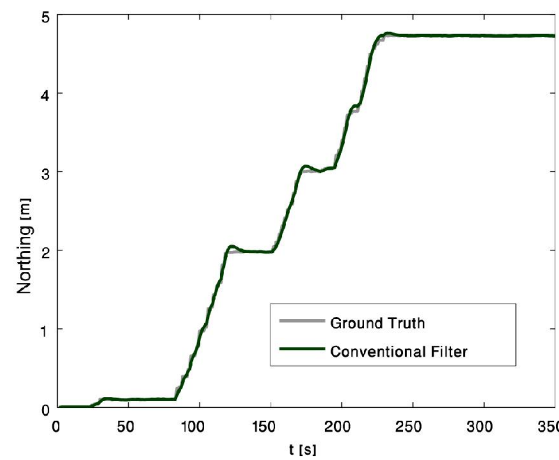

Fig. 7. Estimated path with conventional Kalman filter (σ_2=0.05 m/sec^2)

on model performance,respectively σ_1=0.1 m/s^2 and σ_2=0.05 m/s^2. Figures 5 and 6 show the positioning errors of the conventional Kalman filter with these two dynamic noise levels. It is clearly shown that the positioning errors are significantly different when different dynamic noise levels were selected. The peaks in Fig. 6 are larger than those in Fig. 5. The RMS errors are 50 mm (Easting) and 47 mm (Nothing), and 76 mm (Easting) and 72 mm (Northing) for the noise levels of 0.1 m/s^2 and 0.05 m/s^2, respectively. Figure 7 shows the ground truth path estimated with carrier phase measurements and the estimated one using C/A code pseudorange with conventional Kalman filter. For readability's sake, only the movement of the vehicle along the North-direction is considered. On the straight lines, the positioning errors are similar with different noise levels. However, with tight constraint on the dynamic noise level (0.05 m/s^2), the positioning errors are significantly larger when the rover stops or turns. Afterwards, the same data set was processed using the adaptive filter discussed in Section II. Figure 8 shows the positioning errors with the adaptive Kalman filter. Comparing Fig. 8 with Figs. 5 and 6, it is clearly shown that the positioning errors with the adaptive filter are significantly smaller than the conventional filter. For the fading memory filter, the positioning errors are reduced to 17 mm and 17 mm RMS for Easting and Northing, respectively. The error reduction resulted in 60% and 75% compared with the two conventional filters. Also, the errors in Fig. 8 are more uniformly distributed and this means that the positioning errors are not associated with the sudden manoeuver changes of the vehicle. The advantage of the adaptive approach is also demonstrated in Fig. 9 where the two filter implementations are compared side by side during a sudden stop of the rover occurring between 116 and 140 seconds of the experiment. The improvement in the accuracy of the position estimation is obvious and significant. Table II summarizes the RMS error using conventional and adaptive filtering algorithm. For the adaptive filters, the initial noise levels are chosen the same as the conventional Kalman filter. The dynamic noise

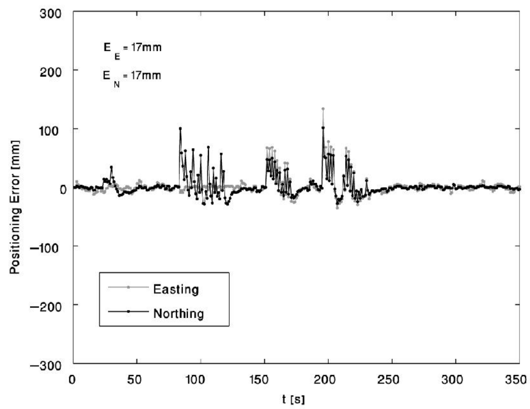

Fig. 8. Positioning error with the adaptive Kalman filter

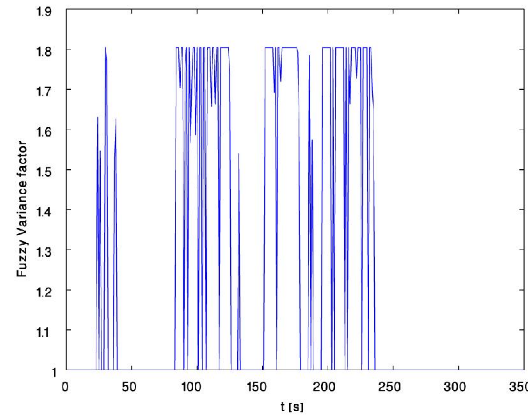

Fig. 10. Output of the DI

level greatly affects the performance of the conventional filter. For fading memory filter, the positioning errors are reduced significantly. A number of other tests have also been performed and the results confirm the above conclusions. Finally, Fig. 10 shows the output of the DI. The adaptive compensation was injected for 31% of the time in this experiment.

B. Closed-path experiment

The performance of the adaptive filter over long distances was assessed by running some closed-path tets. In these experiments, the rover started its at a flat, marked location and, after a near-rectangular-path, returned to the same position, resulting in a total travel distance of D = 150m. The speed of the rover was 35 cm/sec. During its course, the robot descended and then reclimbed up a 20-meter long slope of approximately 16% of inclination. Again, the actual path of the vehicle was derived by GPS RTK technique with accuracy of centimeter level. Then, the conventional filter and the adaptive filter were applied separately to process differential pseudorange data. The paths estimated by the two methods are plotted in Fig.11 and Fig.12 compared with the ground truth data, respectively. Figure 13 shows the altitude estimates of the vehicle during the experiment, instead. It is clear to see that the position accuracy with the conventional kalman filter is much worse than that with the adaptive filter. The peak errors are associated with the turns of trajectory. Table III shows the RMS errors provided by the three filters and confirms the improvement in the positing system accuracy using the fading memory, which allows a six-fold and four-fold reduction in the error, respectively, when compared with the two conventional kalman filters.

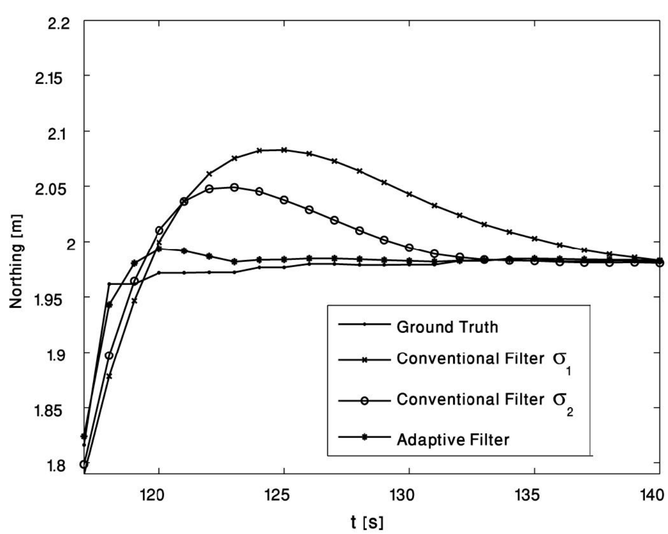

Fig. 9. Overshoot due to a sudden stop of the rover, as estimated by conventional and adaptive Kalman filter

IV. CONCLUSION

Conventional Kalman filter is very sensitive to the selection of the noise level of the dynamic model. This paper presented an adaptive Kalman filter to improve GPS-based localization of mobile robots in outdoor applications. The proposed approach was based on a fuzzy indicator to define a scale factor for the predicted covariance matrix (fading memory) by observing the size of residuals. It was shown that this method is effective in experimental trials using a rough-terrain rover, reducing the positioning error up to 75% than conventional Kalman filters. This technique can be used to improve localization accuracy in rough-terrain autonomous vehicles.

Filter Type	Conventional Kalman Filter σ_1=0.1 m/sec^2	Conventional Kalman Filter σ_2=0.05 m/sec^2	Adaptive Filter
E (mm)	76	50	17
N (mm)	72	47	17

TABLE II

POSITIONING ERRORS WITH DIFFERENT FILTERS, *L*-PATH EXPERIMENT

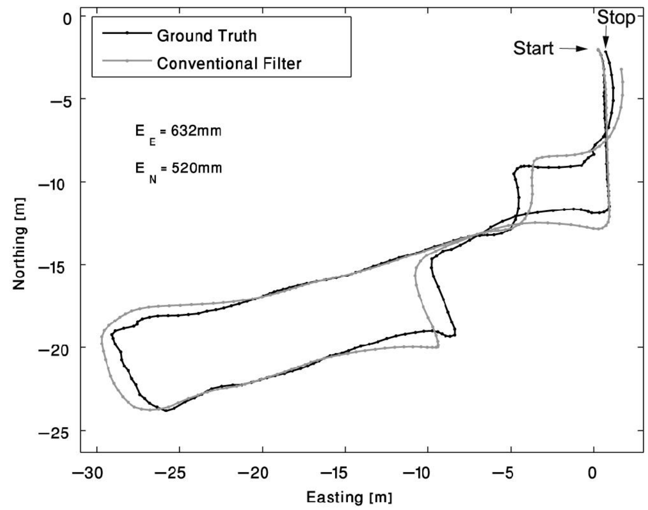

Fig. 11. Path of the rover as estimated by the conventional Kalman filter (σ_1=0.1 m/sec^2)

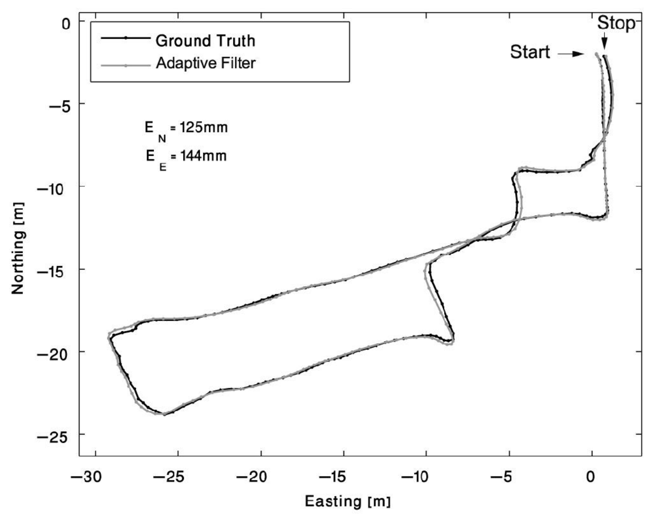

Fig. 12. Path of the rover as estimated by the adaptive Kalman filter

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Filter Type	Conventional Kalman Filter σ_1=0.1 m/sec^2	Conventional Kalman Filter σ_2=0.05 m/sec^2	Adaptive Filter
E (mm)	986	632	125
N (mm)	687	520	144
A (mm)	131	90	35

TABLE III

POSITIONING ERRORS USING DIFFERENT FILTERS, CLOSED-PATH EXPERIMENT

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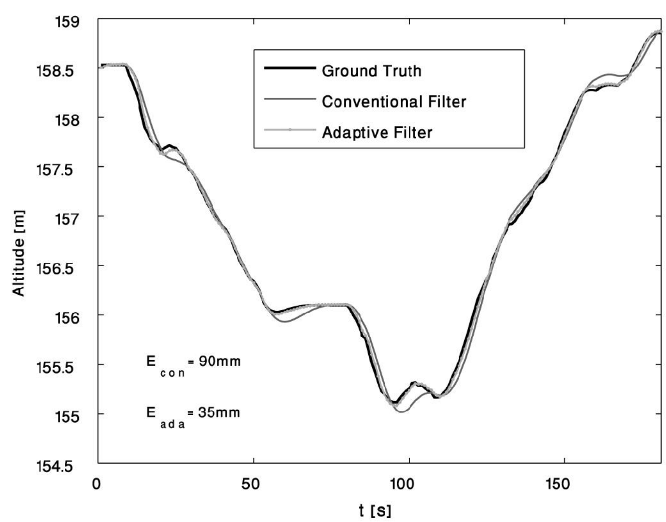

Fig. 13. Altitude estimated by conventional and adaptive Kalman filter